* Can be exact
* RiCatti Equation
* Kirshoff's law

Can be exact

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$M(x,y)\,dx \neq N(x,y)\,dy$$

$$\Rightarrow \frac{M_y - N_x}{M} = g(y)$$

$$\therefore \mu = e^{-\int g(y)\,dy}$$

$$\Rightarrow \frac{M_y - N_x}{N} = f(x)$$

$$\mu = e^{+\int f(x)\,dx}$$

$$\therefore \boxed{M(x,y)\,\mu\,dx + N(x,y)\,\mu\,dy = 0}$$

PAGE
DATE

ex:- $(1-xy)\,dx + (xy - x^2)\,dy = 0$

Solution

$M = (1 - xy)$ $\qquad N = xy - x^2$

$M_y = -x \neq N_x = y - 2x$

$M_y - N_x = -x - y + 2x = x - y = -(y - x)$

$\Rightarrow \mu = \frac{M_y - N_x}{N} = \frac{-\cancel{(y-x)}}{x\cancel{(y-x)}} = \frac{-1}{x}$

$\therefore \Rightarrow \mu = e^{\oplus\int f(x)} = e^{\int(\frac{-1}{x})dx} = \frac{1}{x}$

$\therefore (1 - xy)\frac{1}{x}\,dx + (xy - x^2)\frac{1}{x}\,dy = 0$

$dx\left(\frac{1}{x} - y\right) + (y - x)\,dy = 0$ بالتكامل

بناخذ (1) بس

$\therefore \ln x - yx + (y^2/2) - xy * = C$

$\ln x - yx + \frac{y^2}{2} = C$

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x^2+2yx

ex:- $(4xy+3y^2-x).dx + x(x+2y)\,dy = 0$

متغير

Solution

$M = 4xy+3y^2-x$ $\qquad N = x^2+2yx$

$M_y = (4x+6y) \neq N_x = 2x+2y$

$$f = \frac{M_y - N_x}{N} = \frac{2x+4y}{x(x+2y)} = \frac{2}{x}$$

$$\therefore \mu = e^{+\int f(x)dx} = e^{\int(\frac{2}{x})dx} = e^{2\ln x} = x^2$$

$\mu = x^2$

بالتعويض في المسألة

$\therefore (4x^3y+3y^2x^2-x^3)dx + (x^4+2yx^3)dy = 0$

بالتكامل

$\therefore x^4y + x^3y^2 - \frac{1}{4}x^4 + x^4y + y^2x^3 = c$

$\therefore x^4y + x^3y^2 - \frac{1}{4}x^4 = C$

PAGE
DATE

ex:- $y(x+y+1)\,dx + x(x+3y+2)\,dy = 0$

$$M = xy + y^2 + y \qquad N = x^2 + 3yx + 2x$$

$$M_y = x + 2y + 1 \neq N_x = 2x + 3y + 2$$

$$\mu = \frac{M_y - N_x}{M} = \frac{-x-y-1}{-(-x-y-1)y} = \frac{-1}{y}$$

$$\mu = e^{\ominus\int\left(\frac{-1}{y}\right)dy} = y$$

$$\Rightarrow \therefore (y^2x + y^3 + y^2)\,dx + (x^2y + 3y^2x + 2xy)\,dy = 0$$

$$\left(\frac{y^2x^2}{2} + y^3x + y^2x\right) + \frac{x^2y^2}{2} + y^3x + xy^2 = C$$

$$\therefore \frac{y^2x^2}{2} + y^3x + y^2x = C$$

PAGE
DATE

* Riccatti Equation

$$\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x) \longrightarrow ①$$

$\Rightarrow$ if $A(x)=0$ $\Rightarrow$ linear

$\Rightarrow$ if $B(x)=0$ $\Rightarrow$ can be linear

$\Rightarrow$ if $f(x)$ is a solution for the diff eqn.

$$\Rightarrow y = f(x) + \frac{1}{v(x)}$$

$$\frac{dy}{dx} = \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx}$$ بالتعويض في ①

$$\therefore \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = A\left[f + \frac{1}{v}\right]^2 + B\left[f + \frac{1}{v}\right] + C$$

$$\frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = A\left[f^2 + 2\frac{f}{v} + \frac{1}{v^2}\right] + Bf + \frac{B}{v} + C$$

$$\therefore \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = Af^2 + 2A\frac{f}{v} + \frac{A}{v^2} + Bf + \frac{B}{v} + C$$

$\because f(x)$ الحل الأصلي $\therefore \frac{df}{dx} = Af^2 + Bf + C$

$$\therefore \frac{-1}{v^2}\frac{dv}{dx} = 2A\frac{f}{v} + \frac{A}{v^2} + \frac{B}{v}$$

$$\therefore \frac{-dv}{dx} = 2AfV + A + BV$$

$$\therefore \frac{-dv}{dx} = V(2Af+B) + A$$

$$\therefore \boxed{\frac{dv}{dx} + V(2Af+B) = -A}$$

ex:- $\frac{dy}{dx} = -8xy^2 + 4x(4x+1)y - (8x^3+4x^2-1)$

$\Rightarrow$ if the function $f(x) = x$ is a solution find y ??

{ Solution }

$A = -8x \qquad B = 16x^2+4x \qquad C = -(8x^3+4x^2-1)$

$$\Rightarrow \frac{dv}{dx} + V(2\underset{-8x}{A}\underset{x}{f}+B) = -A$$

$$\frac{dv}{dx} + V(-16x^2 + 16x^2+4x) = +8x$$

$$\frac{dv}{dx} + \underbrace{4xV = 8x}_{\text{linear}} \qquad \Rightarrow I = e^{\int 4x\,dx} = e^{2x^2}$$

$$V = \frac{1}{e^{2x^2}}\left[\int e^{2x^2} * \underset{2(4x)}{8x}\,dx + c\right]$$

تفاضل اسّ الـ e

$$V = \frac{1}{e^{2x^2}}\left[2e^{2x^2} + c\right]$$

$$\therefore y = f + \frac{1}{V_1} = x + \frac{1}{[\;]}$$

Hi . Star

ex:- $\frac{dy}{dx} = (1+x+x^2) - (1+2x)y + y^2$

if $y\, f(x) = x$??? find y

$A = 1$ $\quad B = -(1+2x)$ $\quad C = (1+x+x^2)$

$\Rightarrow \frac{dv}{dx} + v(2Af + B) = -A$

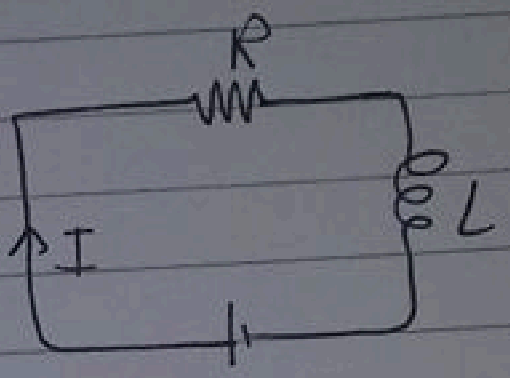
$\frac{dv}{dx} + v(2x + (-1-2x)) = -1$

$\frac{dv}{dx} - v = -1$ $\quad \Rightarrow I = e^{-x}$

$v = \frac{1}{e^{-x}} \left[\int -e^{-x}\, dx + c \right]$

Kirchoff's law

$$E = RI + L\frac{dI}{dt}$$



find I ??

$\div L$

$$\frac{dI}{dt} + \frac{R}{L} = \frac{E}{L}$$ ↘ lineal

$$I = e^{\int (\frac{R}{L})dt} = e^{\frac{R}{L}t} \quad \therefore$$

التيار $\therefore I(t) = \frac{1}{e^{Rt/L}}\left[\int \frac{E}{L} e^{Rt/L} dt + c\right]$

$$I = \frac{1}{e^{Rt/L}}\left[\frac{E}{L}\frac{L}{R} e^{Rt/L} + c\right] = \frac{1}{e^{Rt/L}}\left[\frac{E}{R} e^{Rt/L} + c\right]$$

$$I = \frac{E}{R} + c\,e^{-Rt/L}$$ at $t=0 \rightarrow I(0)=0$

$$\therefore 0 = \frac{E}{R} + c(1) \quad \therefore c = \frac{-E}{R}$$

$$I = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$$

في الميدترم ⇐ ممكن يديلو قيم R و L
ويقول اوجد التيار بعد 0 ثواني
الحل ⇐ هنعوض